EC 1311 TEORIA ELECTROMAGNETICA
FORMULARIO N° 1: ANALISIS DE CAMPOS

Transformación de coordenadas

Rectangulares (x, y, z)	Cilíndricas (ρ, φ, z)	Esféricas (r, θ, φ)
$x=\rho\cos\varphi=r\,\text{sen}\,\theta\cos\varphi$	$\rho=\sqrt{x^2+y^2}=r\,\text{sen}\,\theta$	$r=\sqrt{x^2+y^2+z^2}=\sqrt{\rho^2+z^2}$
$y=\rho\,\text{sen}\,\varphi=r\,\text{sen}\,\theta\,\text{sen}\,\varphi$	$\varphi=\begin{cases}\cos^{-1}\left(x/\sqrt{x^2+y^2}\right), & \text{si } y\geq 0\\ 2\pi-\cos^{-1}\left(x/\sqrt{x^2+y^2}\right), & \text{si } y<0\end{cases}$	$\theta=\cos^{-1}\left(\dfrac{z}{\sqrt{x^2+y^2+z^2}}\right)=\cos^{-1}\left(\dfrac{z}{\sqrt{\rho^2+z^2}}\right)$
$z=r\cos\theta$	$z=r\cos\theta$	$\varphi=\begin{cases}\cos^{-1}\left(x/\sqrt{x^2+y^2}\right), & \text{si } y\geq 0\\ 2\pi-\cos^{-1}\left(x/\sqrt{x^2+y^2}\right), & \text{si } y<0\end{cases}$

Transformación de los vectores unitarios	Elementos diferenciales		
$\mathbf{1}_x=\cos\varphi\mathbf{1}_\rho-\text{sen}\,\varphi\mathbf{1}_\varphi=\text{sen}\,\theta\cos\varphi\mathbf{1}_r+\cos\theta\cos\varphi\mathbf{1}_\theta-\text{sen}\,\varphi\mathbf{1}_\varphi$	$d\mathbf{x}=dx\mathbf{1}_x$	$d\boldsymbol{\rho}=d\rho\mathbf{1}_\rho$	$d\mathbf{r}=dr\mathbf{1}_r$
$\mathbf{1}_y=\text{sen}\,\varphi\mathbf{1}_\rho+\cos\varphi\mathbf{1}_\varphi=\text{sen}\,\theta\,\text{sen}\,\varphi\mathbf{1}_r+\cos\theta\,\text{sen}\,\varphi\mathbf{1}_\theta+\cos\varphi\mathbf{1}_\varphi$	$d\mathbf{y}=dy\mathbf{1}_y$	$d\mathbf{l}_\varphi=\rho d\varphi\mathbf{1}_\varphi$	$d\mathbf{l}_\theta=rd\theta\mathbf{1}_\theta$
$\mathbf{1}_z=\cos\theta\mathbf{1}_r-\text{sen}\,\theta\mathbf{1}_\theta$	$d\mathbf{z}=dz\mathbf{1}_z$	$d\mathbf{z}=dz\mathbf{1}_z$	$d\mathbf{l}_\varphi=r\,\text{sen}\,\theta\, d\varphi\mathbf{1}_\varphi$
$\mathbf{1}_\rho=\cos\varphi\mathbf{1}_x+\text{sen}\,\varphi\mathbf{1}_y=\text{sen}\,\theta\mathbf{1}_r+\cos\theta\mathbf{1}_\theta$	$da_x=dy\,dz$	$da_\rho=\rho d\varphi\,dz$	$da_r=r^2\text{sen}\,\theta\, d\varphi\, d\theta$
$\mathbf{1}_\varphi=-\text{sen}\,\varphi\mathbf{1}_x+\cos\varphi\mathbf{1}_y$	$da_y=dx\,dz$	$da_\varphi=d\rho\,dz$	$da_\theta=r\,\text{sen}\,\theta\, d\rho\, d\varphi$
$\mathbf{1}_r=\text{sen}\,\theta\cos\varphi\mathbf{1}_x+\text{sen}\,\theta\,\text{sen}\,\varphi\mathbf{1}_y+\cos\theta\mathbf{1}_z=\text{sen}\,\theta\mathbf{1}_\rho+\cos\theta\mathbf{1}_z$	$da_z=dx\,dy$	$da_z=\rho\, d\rho\, d\varphi$	$da_\varphi=r\,dr\,d\theta$
$\mathbf{1}_\theta=\cos\theta\cos\varphi\mathbf{1}_x+\cos\theta\,\text{sen}\,\varphi\mathbf{1}_y-\text{sen}\,\theta\mathbf{1}_z=\cos\theta\mathbf{1}_\rho-\text{sen}\,\theta\mathbf{1}_z$	$dV=dx\,dy\,dz$	$dV=\rho\, d\rho\, d\varphi\, dz$	$dV=r^2\text{sen}\,\theta\, dr\, d\varphi\, d\theta$

Gradiente

Coordenadas rectangulares	Coordenadas cilíndricas	Coordenadas esféricas
$\nabla\Phi=\dfrac{\partial\Phi}{\partial x}\mathbf{1}_x+\dfrac{\partial\Phi}{\partial y}\mathbf{1}_y+\dfrac{\partial\Phi}{\partial z}\mathbf{1}_z$	$\nabla\Phi=\dfrac{\partial\Phi}{\partial\rho}\mathbf{1}_\rho+\dfrac{1}{\rho}\dfrac{\partial\Phi}{\partial\varphi}\mathbf{1}_\varphi+\dfrac{\partial\Phi}{\partial z}\mathbf{1}_z$	$\nabla\Phi=\dfrac{\partial\Phi}{\partial r}\mathbf{1}_r+\dfrac{1}{r}\dfrac{\partial\Phi}{\partial\theta}\mathbf{1}_\theta+\dfrac{1}{r\,\text{sen}\,\theta}\dfrac{\partial\Phi}{\partial\varphi}\mathbf{1}_\varphi$

Divergencia

Coordenadas rectangulares	Coordenadas cilíndricas	Coordenadas esféricas
$\nabla\cdot\mathbf{F}=\dfrac{\partial F_x}{\partial x}+\dfrac{\partial F_y}{\partial y}+\dfrac{\partial F_z}{\partial z}$	$\nabla\cdot\mathbf{F}=\dfrac{1}{\rho}\dfrac{\partial(\rho F_\rho)}{\partial\rho}+\dfrac{1}{\rho}\dfrac{\partial F_\varphi}{\partial\varphi}+\dfrac{\partial F_z}{\partial z}$	$\nabla\cdot\mathbf{F}=\dfrac{1}{r^2}\dfrac{\partial(r^2F_r)}{\partial r}+\dfrac{1}{r\,\text{sen}\,\theta}\dfrac{\partial(\text{sen}\,\theta\, F_\theta)}{\partial\theta}+\dfrac{1}{r\,\text{sen}\,\theta}\dfrac{\partial F_\varphi}{\partial\varphi}$

Componentes del rotacional

Coordenadas rectangulares	Coordenadas cilíndricas	Coordenadas esféricas
$(\nabla\times\mathbf{F})_x=\dfrac{\partial F_z}{\partial y}-\dfrac{\partial F_y}{\partial z}$	$(\nabla\times\mathbf{F})_\rho=\dfrac{1}{\rho}\dfrac{\partial F_z}{\partial\varphi}-\dfrac{\partial F_\varphi}{\partial z}$	$(\nabla\times\mathbf{F})_r=\dfrac{1}{r\,\text{sen}\,\theta}\dfrac{\partial(\text{sen}\,\theta\, F_\varphi)}{\partial\theta}-\dfrac{1}{r\,\text{sen}\,\theta}\dfrac{\partial(F_\theta)}{\partial\varphi}$
$(\nabla\times\mathbf{F})_y=\dfrac{\partial F_x}{\partial z}-\dfrac{\partial F_z}{\partial x}$	$(\nabla\times\mathbf{F})_\varphi=\dfrac{\partial F_\rho}{\partial z}-\dfrac{\partial F_z}{\partial\rho}$	$(\nabla\times\mathbf{F})_\theta=\dfrac{1}{r\,\text{sen}\,\theta}\dfrac{\partial F_r}{\partial\varphi}-\dfrac{1}{r}\dfrac{\partial(rF_\varphi)}{\partial r}$
$(\nabla\times\mathbf{F})_z=\dfrac{\partial F_y}{\partial x}-\dfrac{\partial F_x}{\partial y}$	$(\nabla\times\mathbf{F})_z=\dfrac{1}{\rho}\dfrac{\partial(\rho F_\varphi)}{\partial\rho}-\dfrac{1}{\rho}\dfrac{\partial F_\rho}{\partial\varphi}$	$(\nabla\times\mathbf{F})_\varphi=\dfrac{1}{r}\dfrac{\partial(rF_\theta)}{\partial r}-\dfrac{1}{r}\dfrac{\partial(F_r)}{\partial\theta}$

Laplaciano ($\nabla^2\Phi$)

Coordenadas rectangulares	Coordenadas cilíndricas	Coordenadas esféricas
$\dfrac{\partial^2\Phi}{\partial x^2}+\dfrac{\partial^2\Phi}{\partial y^2}+\dfrac{\partial^2\Phi}{\partial z^2}$	$\dfrac{1}{\rho}\dfrac{\partial}{\partial\rho}\left(\rho\dfrac{\partial\Phi}{\partial\rho}\right)+\dfrac{1}{\rho^2}\dfrac{\partial^2\Phi}{\partial\varphi^2}+\dfrac{\partial^2\Phi}{\partial z^2}$	$\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial\Phi}{\partial r}\right)+\dfrac{1}{r^2 sen\,\theta}\dfrac{\partial}{\partial\theta}\left(sen\,\theta\dfrac{\partial\Phi}{\partial\theta}\right)+\dfrac{1}{r^2 sen^2\,\theta}\dfrac{\partial^2\Phi}{\partial\varphi^2}$